Bayesian Posterior Prediction and Meta-Analysis:

An Application to the Value of Travel Time Savings

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In the evaluation of transportation infrastructure projects, some non-tradable goods such as time are usually key determinants of the result. However, obtaining monetary values for these goods is not always easy. For this purpose, this paper presents an approach that combines Bayesian posterior prediction and meta-analysis. This methodology will allow obtaining predictive distributions of the monetary values for this type of goods. Therefore, uncertainty is formally considered in the analysis. Moreover, the proposed method is easy to apply and inexpensive both in terms of time and money. Finally, an illustrative application to the value of travel time savings is also presented.

Keywords: Cost-Benefit Analysis, Bayesian Prediction, Meta-Analysis, Value of Travel Time Savings.

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1.- Introduction

The Value of Travel Time refers to the cost of time spent on transport, including waiting as well as the actual travel. It includes costs to consumers of personal (unpaid) time spent on travel, and costs to businesses of paid employee time spent in travel. Travel time is one of the largest categories of transport costs. Hence, time savings are often the greatest benefit of transport projects such as new and expanded roadways, and public transit improvements. The Value of Travel Time Savings -henceforth VTTS- refers to the benefits from reduced travel time. As previously stated, in most of the transportation projects travel time savings represent the most important source of social benefits. Therefore, VTTS is considered a key determinant of the evaluation results.

On the other hand, there are other important sources of user benefits that are based on non-tradable goods. The value of a statistical life -henceforth VSL- is the best example in this category. Public choices about safety in a democratic society require estimates of the willingness of people to trade off wealth for a reduction in the probability of death. Estimates of these trade-offs are used in evaluating environmental issues, public safety in travel, medical interventions and in many other areas. It has become common to call this trade-off the value of a statistical life (VSL). VSL is therefore a measure that is widely used for the evaluation of transportation infrastructure projects that, among other things, reduce the risk of fatal accidents. The literature on the estimation of VSL across the world is enormous, Viscusi and Aldy (2003) provide a good survey.

In order to estimate the VTTS (or the VSL) for a particular project, the most desirable option is to conduct a specific study among the potential users of the "project". The project-specific methodologies usually adopted can be classified as either revealed preference or stated preference. The main difference between these two types of methodologies is given by the fact that revealed preferences studies take into account the actual behavior of persons or firms while stated preferences studies try to investigate the preferences of economic actors among alternatives which are only hypothetical (see Zamparini and Reggiani (2007b) for more details on both approaches). However, all project-specific studies have a common characteristic: they are always very expensive both in terms of time and money. Therefore, in practice, the usual approach when evaluating a particular project is to determine the VTTS (or VSL) according to recommended values at a national level (see for example Nellthorp et al. (2001)) or by simply imputing the values obtained in previous case-specific studies for similar projects. In any event, previous Cost Benefit Analysis (CBA) studies are always based on a single VTTS (or VSL) value. As a consequence, uncertainty is not taken into consideration. The imputed VTTS (or VSL) is considered as the true value without any doubt (i.e. all the calculations are carried out by assuming that this single value is unambiguously true). However, in reality, this imputed value is only an estimate, and uncertainty is inherent in any estimation procedure.

The main object of interest in the CBA is the Net Present Value (NPV) of a particular project under evaluation. Moreover, since the role of risk and uncertainty in the CBA is crucial (e.g. Layard and Glaister (1994)), for instance different demand scenarios are always analyzed given the uncertainty associated with demand forecasting, practitioners usually consider the probability distribution of NPV's rather than a single value. Therefore, as pointed out by de Rus (2008) among others, the existence of uncertainty when imputing both VTTS and VSL should also be considered in the construction of the NPV's distribution.

Against this background, this paper proposes to use a meta-analytical approach that can be easily applied to any project under evaluation. The method is meta-analytical since it will review previous scientific studies. Meta-analysis represents an appropriate technique in order to study VTTS or VSL, given that it can summarize the results of very heterogeneous studies. Meta-analytical techniques have been previously considered in the literature, for example Zamparini and Reggiani (2007a) use a meta-analytical approach to study how different project characteristics affect the VTTS and Miller (2000) estimates the effect of project characteristics on the VSL. However, all previous meta-analytical studies on VTTS (or VSL) are based on frequentist approaches in which a single value can be predicted with some ad-hoc uncertainty measures. Given the use of Bayesian methods, the approach considered in this paper will allow to obtain the whole predictive distribution of the VTTS (or VSL) obtained in previous studies conditional on project-specific characteristics. By doing so, uncertainty is formally considered and can be easily incorporated to the NPV's distribution.

The remainder of the paper is organized as follows. Section 2 describes the approach. Firstly I explain Bayesian posterior prediction, then I briefly summarize the concept of meta-analysis and I introduce how to combine both methods. In Section 3 I present an example of how the methodology can work in practice. In particular, I show how we can obtain the distribution of VTTS for a particular project under evaluation. The final section concludes.

2.- Methodology

Since the approach I present in this paper is a combination of two different techniques, I will proceed to explain both concepts separately. Then, I will present how they are combined.

2.1.- Bayesian Posterior Prediction

Bayesian econometrics is the systematic use of a result from elementary probability, Bayes' theorem. Suppose we have a model given by $f_y(y;\theta)$, where y represents the data and θ the parameters. The object of interest from an econometric perspective is the vector of parameters θ. The logic of Bayesian inference is to apply Bayes' theorem such that:

$$p(\theta \mid y) \propto p(y \mid \theta)\, p(\theta) \qquad (1)$$

where $p(\theta \mid y)$ is referred to as the *posterior density*, $p(y \mid \theta)$ is the *likelihood* function of the data given the parameters and $p(\theta)$ is the *prior density* of the parameters. In the present case, like in most econometrics, prediction is a major concern. That is, given the observed data, y, the econometrician may be interested in predicting some unobserved data y^*. In our case, the observed data

y will be the different estimates of the VTTS (or VSL) for projects of previous studies and their characteristics. The unobserved data y^* that we want to predict will be the VTTS (or VSL) for new projects under evaluation.

The Bayesian reasoning argues that uncertainty about the unobserved elements (y^*) is summarized by a conditional probability statement. That is, prediction should be based on the posterior predictive density $p(y^* | y)$. This marginal density can be obtained from the joint density of y^* and θ through integration:

$$p(y^* | y) = \int p(y^*, \theta | y) d\theta$$

Moreover, by the law of total probablity:

$$p(y^*, \theta | y) = p(y^* | y, \theta) p(\theta | y)$$

So that the predictive distribution of interest becomes:

$$p(y^* | y) = \int p(y^* | y, \theta) p(\theta | y) d\theta \qquad (2)$$

The predictions we would obtain by applying Bayesian posterior prediction[2] are in a different spirit of those obtained by classical methods. The important difference is that with the Bayesian approach, we predict the whole distribution of unobserved data instead of a single data point with some standard error. This is the main advantage of the approach presented in this paper with regard to the treatment of uncertainty.

2.2.- Meta-Analysis

Meta-analysis is defined as the process or technique of synthesizing research results by using various statistical methods to retrieve, select, and combine results from previous separate but related studies. The basic purpose of meta-analysis is to provide the same methodological rigor to a literature review that we require from experimental research.

Among the advantages of meta-analysis over classical literature reviews or

[2] For more details about the Bayesian methods presented here refer to Lancaster (2004).

single empirical studies we can mention: (i) its ability to control for between-study variation; (ii) its higher statistical power than a single study included in the meta-analysis; (iii) it can include extra variables to explain variation; (iv) it imposes a discipline on the process of summing up research findings; (v) it is capable of finding relationships across studies that are obscured otherwise.

On the other hand, the steps required for conducting a meta-analytical study can be summarized as follows: (1) define the meta-analytic research question; (2) locate the relevant literature; (3) combine and make homogeneous the data from the relevant literature; (4) analyze the resulting meta-analytic database; (5) report and interpret the results.

The focus of this paper is the meta-analysis of different VTTS estimates in order to find the main determinants of VTTS that allow us to make out-of-sample forecasts of VTTS (step 1). As mentioned in the introduction, the literature on VTTS is enormous, so that we will need to select the appropriate studies to be considered (step 2). Thus, once we have compiled and combined enough data of different VTTS studies (step 3), we will be able to carry out a meta-analysis using Bayesian posterior prediction (step 4) in order to summarize all the information in a rigorous statistical manner and obtain the VTTS predictive distribution of interest (step 5).

A discussion of the huge literature on meta-analysis is outside the scope of this paper. Hunter and Schmidt (2004) is a good reference for those readers interested in meta-analysis *per se*.

2.3.- Meta-Analytical Bayesian Posterior Prediction

The combination of the two techniques described above allows to obtain the whole distribution of the monetary values of non-tradable goods such as time. The resultant procedure can be denominated Meta-Analytical Bayesian Posterior Prediction.

The word prediction refers to the fact that we will obtain (or forecast) non-observed values. That is to say, based on a sample of observed VTTS (or VSL) from previous analyses, we will be able to predict the VTTS (or VSL) for new projects under evaluation that had never been analyzed before. The method also uses Bayesian methods that will allow us to incorporate uncertainty from the very beginning in a more natural way than classical approaches. Finally, given the compilation of data from previous studies and the use of statistical techniques in order to combine this information, the approach is also called meta-analytical.

I next turn to formally introduce the methodology. We depart from a linear regression model:

$$y_i = x_i'\beta + v_i \qquad (3)$$

where $i = 1,...,N$ refers to the N previous studies for which we have data. y_i is the value of the non-tradable good (i.e. VTTS or VSL) reported in the study i and x_i is the $k \times 1$ vector of observable characteristics of study i (for example trip purpose or mode in the case of VTTS). By stacking the N observations in vectors we can rewrite (3) in matrix form:

$$Y = X\beta + V \qquad (4)$$

where now, Y and X are a $N \times 1$ vector and a $N \times k$ matrix of data respectively. β is a $k \times 1$ vector of parameters and V is the $N \times 1$ vector of disturbance terms. Given the model in (4), we can now turn to the application of Bayesian posterior prediction. For this purpose we follow a sequential procedure based on three steps: (i) Elicitation of the likelihood function for the data and the prior distribution for the parameters. (ii) Given the likelihood and the prior, obtain the posterior distribution of the parameters, and finally, (iii) solve the integral in (2) in order to obtain the posterior predictive distribution.

First of all, I propose to assume that the error term in (4) follows a multivariate normal distribution with zero mean and variance-covariance matrix given by $\sigma^2 I_N$:

$$V \sim N(0, \sigma^2 I_N)$$

where I_N is the identity matrix of order N (i.e. a diagonal matrix with ones in the main diagonal) so that v_i's are iid $N(0,\sigma^2)$ for $i=1,...,N$.

The previous assumption implies that the likelihood is given by:

$$p(Y\mid\beta,\sigma^2)=\frac{\sigma^{-\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}}\left\{\exp\left[-\frac{1}{2\sigma^2}(Y-X\beta)^{'}(Y-X\beta)\right]\right\} \tag{5}$$

In many situations, when we face the problem of choosing a prior distribution for the parameters of a model, we have very little (if any) prior information for such task. This is also the situation we are facing here, and therefore I propose to assume a noninformative (or diffuse) prior for the parameters. As the likelihood function in (5) belongs to the family of normal-gamma distributions, I will elicit a diffuse prior for the parameters by assuming a normal-gamma distribution[3] with infinite variance. As the variance is a measure of uncertainty, by fixing it to infinity, we are assuming that we do not have any prior information. In particular, the normal-gamma prior can be written as follows:

$$\beta,\sigma^2\sim NG(\underline{\beta},\underline{\Sigma},\underline{s}^{-2},\underline{\nu})$$

where $\underline{\beta},\underline{\Sigma},\underline{s}^{-2}$ and $\underline{\nu}$ are prior hyperparameters to be fixed by the researcher.

Given the likelihood and prior proposed above, and by using Bayes' theorem as in (1), the posterior distribution of the parameters is:[4]

$$\beta,\sigma^2\mid y\sim NG\left(\overline{\beta},\overline{\Sigma},\overline{s}^{-2},\overline{\nu}\right) \tag{6}$$

where $\overline{\beta},\overline{\Sigma},\overline{s}^{-2}$ and $\overline{\nu}$ characterize the posterior distribution and are given by:

$$\overline{\Sigma}=(\underline{\Sigma}^{-1}+X'X)^{-1}$$

$$\overline{\beta}=\overline{\Sigma}(\underline{\Sigma}^{-1}\underline{\beta}+X'X\widehat{\beta}_{OLS})$$

$$\overline{\nu}=\overline{\nu}+N$$

where $\widehat{\beta}_{OLS}=(X'X)^{-1}X'Y$ and $\overline{s}^{-2}$ is defined implicitly through:

$$\overline{\nu s}^{2}=\underline{\nu s}^{2}+\nu s^2+(\widehat{\beta}_{OLS}-\underline{\beta})'[\underline{\Sigma}+(X'X)^{-1}]^{-1}(\widehat{\beta}_{OLS}-\underline{\beta})$$

[3] A prior belonging to the same family of distributions as the likelihood is a natural conjugate prior.
[4] See Koop (2003) for a proof of this result.

where:

$$s^2 = \frac{(Y - X\widehat{\beta}_{OLS})'(Y - X\widehat{\beta}_{OLS})}{N-k}$$

Given this prior and posterior distributions, we can create a purely noninformative prior by choosing $\underline{\nu} = 0$ and setting $\underline{V}^{-1}$ to a small value. One possibility is to set $\underline{V}^{-1} = cI_k$, where c is a scalar, and then let c go to zero. This would be equivalent to eliciting a prior distribution with infinite variance so that it becomes noninformative. By doing so, we find that the parameters $\overline{\beta}, \overline{\Sigma}, \overline{s}^{-2}$ and $\overline{\nu}$ characterizing the posterior distribution are now:

$$\overline{\Sigma} = (X'X)^{-1}$$

$$\overline{\beta} = \widehat{\beta}_{OLS}$$

$$\overline{\nu} = N$$

$$\overline{s}^{2} = \frac{N-k}{N} s^2$$

Combining the normal-gamma posterior with parameters $\overline{\beta}, \overline{\Sigma}, \overline{s}^{-2}$ and $\overline{\nu}$ with the likelihood function in (5), we are now ready to obtain the predictive distribution of our interest by solving the integral in (2).[5] Given this likelihood and posterior, the integral can be solved analytically (see Zellner (1971) pp. 72-75 for a proof of this result). Moreover, the resultant predictive distribution is a Student's t-distribution defined by the following parameters:

$$y^* \mid y \sim t\left(X^{*'}\overline{\beta}, \overline{s}^{2}\left[1 + X^{*}\overline{\Sigma}X^{*'}\right], \overline{\nu}\right) \qquad (7)$$

where X^* is the $k \times 1$ vector of characteristics of the project under evaluation. Therefore, by simply compiling some information about our project (the X^* vector) and applying (7), we will easily obtain the predictive distribution for the monetary values of the non-tradable goods we are interested in. In the application we will see the kind of information that we will need depending on the availability

[5] Note that in this case the parameters of the model are $\theta = (\beta, \sigma^2)$.

and the value of interest.

2.4.- Truncation of the Predictive Distribution

In the previous subsection we have obtained the predictive distribution for monetary values of non-tradable goods such as time. However, up to this point we have only made use of the rules of mathematics and probability without any economics. For example, the resultant t-distribution has support in all the real line. Thus, this result would imply that we assign some probability mass to negative values of the travel time savings. This is an awkward property of the method that can be easily solved by truncating the distribution.

From previous studies we know that the VTTS (or the VSL) must be between some limits. For example it can not take a negative value in any case. Therefore, for solving this problem, I propose to work with the truncated version of the predictive distribution in (7). The truncated distribution of a random variable x between two points a and b is defined by:

$$f(x \mid a < x < b) = \begin{cases} \dfrac{f(x)}{F(b) - F(a)} & \text{if } a < x < b \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where $F(\cdot)$ is the cumulative distribution function (cdf) of $f(\cdot)$.

Given the above, once we have obtained the parameters of the predictive t-distribution of interest, we will also need to compute its truncated version between the support points with some economic sense from previous studies, for instance, between the maximum and the minimum of all the values previously estimated.

2.5.- Evaluating Predictive Ability

Prediction occupies a central position in the methodology proposed in this paper; hence, evaluating predictive ability is a fundamental concern. Reviews of the forecast evaluation literature, such as Diebold and Lopez (1996), reveal that most attention has been paid to evaluating point forecast. However there is an insightful evaluation method for density forecast. Based on the probability integral transform introduced by Rosenblatt (1952), I propose here to evaluate our predictive distributions in practice by employing the approach proposed by Diebold et al. (1998).

For a presentation of the method, let us start by considering the following ingredients:

1. A sequence of observations $\{y_i\}_{i=1}^N$ generated by the functions $\{f_i(y_i) \mid \Omega_{-i}\}_{i=1}^N$ where $\Omega_{-i} = y_1, ..., y_{i-1}, y_{i+1}, ... y_N$. $\{y_i\}_{i=1}^N$ represents a sample of observed realizations of a random variable, $f_i(y_i)$ is the unknown density function of this random variable evaluated at y_i, and Ω_{-i} is the information set that contains all the y's realizations excluding y_i.

2. A group of predictive densities for these observations $\{p_i(y_i)\}_{i=1}^N$ obtained by using the meta-analytical Bayesian posterior prediction proposed in this paper. The $p_i(\cdot)$ function is just an estimation of the real (and unknown) density function $f_i(\cdot)$.

3. The probability integral transform of the predictive densitiy $p_i(y_i)$ that is given by (see Rosenblatt (1952)):

$$z_i = \int_{-\infty}^{y_i} p_i(u)du = P_i(y_i)$$

The idea of the evaluation method is simple: if the sequence of predictive densities ($\{p_i(y_i)\}_{i=1}^N$) coincides with the sequence of the functions that generated the data ($\{f_i(y_i) \mid \Omega_{-i}\}_{i=1}^N$), that is to say, if the predictive (or estimated) densities coincide with the real densities, then, the sequence of probability integral transforms must satisfy:

$$\{z_i\}_{i=1}^N \overset{\text{iid}}{\sim} U(0,1) \qquad (9)$$

If equation (9) is satisfied, this implies that the density estimates are close enough to the real densities, so that the estimation procedure (Bayesian posterior prediction in our case) provides predictive densities that produce reliable out-of-sample forecasts, that are the main objective of the approach outlined in this paper.

Therefore, in practice we only need to obtain the predictive density for an existent project by using the information in all the other projects and excluding its own information. This means that the matrix of data X that we will employ in the computation of the parameters of the posterior distribution given by (7) will have $N-1$ rows instead of N, and $\bar{\nu}$ will be equal to $N-1$. If we obtain in this way the predictive distribution for all the N projects in our sample, we can then compute the sequence of N probability integral transforms and test the hypothesis in (9) by means of, for instance, a Kolmogorov-Smirnov test.

3.- Application

3.1.- A Fictitious Evaluation

The Council of the European Union through its Council Directive 96/48/EC defines High Speed Rail (HSR) as systems of rolling stock and infrastructure which regularly operate at or above 250 km/h (155 mph) on new tracks, or 200 km/h (124 mph) on existing tracks. As pointed out by Campos et al. (2007), HSR is currently considered as one of the most important technological breakthroughs in passenger transportation developed in the second half of the 20^{th} century.

Much of the technology behind high-speed rail is an improved application of mature standard gauge rail technology using overhead electrification. By building a new rail infrastructure with 20^{th} century engineering, including elimination of constrictions such as roadway at-grade (level) crossings, frequent stops, a succession of curves and reverse curves, and not sharing the right-of-way with freight or slower passenger trains, higher speeds around 250-320 km/h (155-198 mph) are maintained.

The world's first contemporary high volume capable (initially 12 car maximum) "high-speed train" was Japan's Tōkaidō Shinkansen, which officially opened in October 1964. In Europe, high-speed rail started during the International Transport Fair in Munich in June 1965, when a total of 347 demonstration trains were hauled at 200 km/h (124 mph) between Munich and Augsburg. The first regular service at this speed was the TEE "Le Capitole" between Paris and Toulouse in 1967.

In Spain, Alta Velocidad Española (AVE),[6] a service of high speed trains operating at speeds of up to 300 km/h (186 mph), inaugurated its first line between Madrid and Seville in 1992. The Madrid–Zaragoza–Barcelona line was inaugurated on February 2008 after parts of the line had operated since 2003 (Madrid–Zaragoza–Lleida) and 2006 (Lleida–Tarragona), and The Madrid–Segovia–Valladolid line was inaugurated on December 2007.

The Spanish government has an ambitious plan to make 7,000 kilometres (4,300 mi) of high-speed railway with all provincial capitals at most only 4 hours from Madrid, and 6½ hours from Barcelona. According to the Strategic Plan for railway infrastructures developed by the Spanish Ministerio de Fomento Ministry of Public Works, called PEIT, a second expansion programme is planned to start in 2011, when the last lines of the first programme still under construction begin operation. This plan has a ten-year scope, ending 2020, and its ambition is to make the 300 km/h (186 mph) network reach 10,000 kilometres (6,200 mi) by the end of that year. This would be the most extensive network in Europe, with several operational links with France and Portugal and is by far the most ambitious high speed rail plan in the European Union.

One particular line included in this plan is the high speed rail line connecting Asturias with Madrid. This line includes the Variante de Pajares in the Cantabrian Mountains (a mountain range which extends for more than approximately 300 km

[6] The name is literally translated from Spanish as "Spanish High Speed", but also a play on the word ave, meaning "bird".

(180 miles) across northern Spain) so that an expensive 24,667 m-long railway tunnel is required for making this line operational. Given its engineering complexity and its huge costs, many people in Spain is skeptical about the social profitability of this line.

Imagine that a group of researchers is evaluating this line using Cost Benefit Analysis (CBA). Furthermore, imagine that after analyzing the cost of building the infrastructure, the operating and maintenance costs and the current demand and its projections, we have now to analyze user's costs. Given the speed of the HSR, the potential users of the Madrid-Asturias line will save around two hours in every trip if they take the HSR instead of the bus or the car. Therefore, as discussed in the introduction, this savings in travel time are one of the most important sources of profits among the users of the HSR line. In fact, this is the main objective of every single HSR project, to reduce travel time required to connect two different cities. However, in order to quantify this crucial part of the social benefits, we need to assign a monetary value for the travel time savings of all potential users of the HSR between Madrid and Asturias within the whole CBA analysis.

Given the uncertainty associated to the process of estimating this travel time monetary values, the question is: what is the distribution of monetary VTTS that can be imputed for those users? The meta-analytical Bayesian posterior prediction method described in this paper will allow us to answer this relevant question.

3.2.- Meta-Analytical Data

We have data of 90 different studies on VTTS carried out in 15 different countries. In particular we take the VTTS reported in each of the studies together with the characteristics of the study (country, year, trip purpose and mode). This data has been compiled by Zamparini and Reggiani (2007a) and more details about its compilation as well as some descriptive statistics can be found in the original source.[7] It is important to note that VTTS values are all measured as a percentage of the hourly wage in order to facilitate comparisons between all of the sampled studies.

In order to complete the dataset and base our analysis in a richer source of information, I have also collected country-level data on per capita GDP in constant prices (2000 USD) from Penn World Tables 6.2.[8] Per capita GDP in a country will obviously affect the VTTS of that country. It is reasonable to think that in a rich country such as USA, the opportunity cost of its population is high and then monetary value of its inhabitants' time will be higher than in a poorer country like Lesotho. Descriptive statistics of per capita GDP in thousands of 2000 USD collected for the 15 countries in the sample at the time the studies were conducted are shown in Table 1. As expected, the pattern is the same as in Zamparini and Reggiani (2007a). The lowest per capita GDP in the sample corresponds to a study carried out in the United Kingdom in 1959, while the highest per capita GDP refers to a study conducted in Norway in 1997.

All of the variables described above are considered as determinants of the VTTS for a given study. Moreover, all of this information seems to be easy to collect for a given project in a given country.[9] Therefore, given the approach, we only need to collect this information in order to obtain the predictive density distribution of the VTTS for a particular project under evaluation.

3.3.- The VTTS Predictive Distribution

Table 2 shows the main HSR project characteristics we need for estimating the VTTS predictive distribution according to the proposed method. In particular, project characteristics presented in Table 2 comprise the vector X^* in (7), and

[7] A brief summary of the employed variables can be found in Table 3 at the end of this paper.

[8] I decided to use updated data on GDP per capita from PWT 6.2 at 2000 prices instead of the World Bank data used in Zamparini and Reggiani (2007a) at 1995 prices.

[9] Note that these variables would comprise the vector X^* in (7) and they would allow us to obtain the distribution of $VTTS^*$.

they include the country, the year, the mode of transport, the trip purpose[10] and per capita GDP. Given this variables and the estimated posterior parameters, the predictive t-distribution in (7) is defined by:

$$\text{Mean} = X^{*'}\overline{\beta} = 130.87$$

$$\text{Variance} = \overline{s}^{2}\left(1 + X^{*}\overline{\Sigma}X^{*'}\right) = 2976.22$$

$$\text{Degrees of freedom} = \overline{\nu} = 90$$

Given the above, if we call $VTTS^{*}$ the unobserved value of travel time savings for the HSR line between Madrid and Asturias, our posterior predictive density (obtained with the method proposed in this paper) is a truncated t-distribution with mean 130.87, variance 2976.22, degrees of freedom 90 and truncation points 13 and 342 given by the minimum and the maximum VTTS of all the available previous studies. This predictive distribution contains very rich and relevant information about the VTTS for the potential passengers of the Madrid – Asturias HSR line. For instance, among the passengers travelling for bussiness, the average monetary value of an hour saved by the new HSR line is 130.87% of the hourly wage, which is around 23 USD,[11] so 46 USD is the estimated average VTTS per trip and bussiness passenger. If we compute the predictive distribution for the case of leisure trips, the mean of the VTTS distribution is now 49.21 so that the average VTTS of an hour saved by the new HSR line when travelling for leisure is 49.21% of the hourly wage. This implies an average VTTS per hour and leisure passenger of around 8.75 USD, which is substantially smaller than the case of bussiness trips (23 USD) as expected.

Moreover, having estimated the whole posterior, we can also compute many different quantities of interest such as the probability of the VTTS being larger than the hourly wage rate, which is 73.5% when the purpose of travel is bussiness and

[10] As a baseline scenario, I will estimate the distribution for those users whose trip purpose is employer's business. However, as we will see, the approach can be easily extended to estimate the VTTS for other trip purposes.

[11] Assuming a total of 1688 hours worked per year and given per capita GDP in Table 2, the average Spanish hourly wage is around 17.75 USD.

24.2% if the purpose is leisure.

For additional insights, we can also have a look to the graph of the predictive distributions in Figure 1 and Figure 2 which makes it clear how the VTTS is distributed in the HSR between Madrid and Oviedo when the purpose of travel is bussiness or leisure respectively. This example makes clear the advantages of estimating the whole distribution with respect to estimating (or using) a single VTTS value.

On the other hand, this posterior predictive distribution is an ingredient of paramount importance in the CBA analysis of the Madrid-Asturias HSR line. In particular, it is only necessary to simulate the different scenarios in our CBA according to this distribution[12] for the VTTS in order to get the NPV's distribution of interest.

I now turn to evaluate the predictive ability of the method in this particular context. Firstly, I obtain the VTTS predictive distribution for all the 90 studies in the dataset using the information available in the remaining 89 studies. Then, I compute the probability integral transform of all the 90 densities. In this way, I have a sequence $\{z_i\}_{i=1}^{90}$ and I have to test whether this sequence is independent and identically distributed as a $U(0,1)$ distribution. For this purpose, I employ a Kolmogorov-Smirnov test whose null hypothesis is that the sequence is independent and identically distributed as a $U(0,1)$ distribution. The obtained p-values are 0.80 and 0.69 repectively for the bussiness and the leisure cases, hence we cannot reject the null in any case. Therefore, given this result, we are able to conclude that the predictive ability of the method is satisfactory, at least in this particular application.

4.- Concluding Remarks

In the evaluation of transportation infrastructure projects, some non-tradable

[12] A method of simulating data from a truncated distribution is described in the Appendix A.1. at the end of this paper.

goods such as time are usually key determinants of the result. An important challenge in Cost Benefit Analysis (CBA) is how to assign monetary values to this type of goods in order to calculate the Net Present Value (NPV) of the project.

This paper introduces a new methodology that allows assigning monetary values to non-tradable goods such as lives or time. Moreover, the method allows obtaining the whole probability distribution of the monetary values rather than a single number, which is an additional benefit of the proposed approach.

More concretely, the methodology is labeled as meta-analytical Bayesian posterior prediction because it combines Bayesian prediction and meta-analysis. I think the approach is appealing because it is very easy to apply, it requires little information, and it is inexpensive both in terms of time and money. Moreover, the predictive ability of the approach can be easily tested as shown in the paper.

In order to illustrate how the method works in practice, a case study is also presented in the paper. In the framework of the Strategic Plan for railway infrastructures developed by the Spanish Ministerio de Fomento Ministry of Public Works, called PEIT, there is a projected High Speed Rail (HSR) line that connects Madrid and Asturias. One of the main objectives of every HSR project is to reduce the travel time required to connect two different points. However, in order to quantify the benefits emerging from the reduction in travel time we need to assign monetary values to the travel time savings of all potential users of the line. Using the method proposed in the paper we estimate the distribution of monetary values for the VTTS in this particular line, and obtain, for instance, that the estimated average VTTS per hour and passenger is around 23 USD if the purpose of the trip is bussiness and 8.75 USD if the trip purpose is leisure.

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A. Appendix

A.1. Random Draws from a Truncated Density

Given a random variable $x \sim f$, I next explain how to extract random draws in practice from its truncated density $f(x \mid a < x < b)$.

First of all, the truncated cumulative density function (cdf) of x is given by:

$$\frac{F(x) - F(a)}{F(b) - F(a)}$$

We know that the cdf of a random variable is always between 0 and 1 and is uniformly distributed. Therefore, we can generate the random draws from the truncated density as follows:

$$u \sim U[0,1]$$

$$x = F^{-1}[uF(b) + (1-u)F(a)]$$

If the variable is not standarized with mean μ and variance σ^2, we would simulate as follows:

$$x = \mu + \sigma F^{-1}[uF(b) + (1-u)F(a)]$$

Given the above, in practice we only need to simulate a sequence of numbers from a $U(0,1)$ distribution, and then apply the previous formulas.

References

1. Campos, J., de Rus, G., Barron, I., (2007) Some stylized facts about high speed rail. A review of HSR experiences around the world. Unpublished Manuscript.
2. de Rus, G., (2008) Analisis Coste-Beneficio, Ariel Economia.
3. Diebold, F., Gunther, T., Tay, A., (1998) Evaluating density forecasts with applications to financial risk management. *International Economic Review*, 39, 863-883.
4. Diebold, F., Lopez, J., (1996) Forecast evaluation and combination. *Handbook of statistics*, 241-268.
5. Hunter, J., Schmidt, F., (2004) Methods of Meta-analysis: Correcting error and bias in research findings, SAGE.
6. Koop, G., (2003) Bayesian Econometrics, Wiley-Interscience.
7. Lancaster, T., (2004) An Introduction to Modern Bayesian Econometrics, Blackwell Publishing.
8. Layard, R., Glaister, S., (eds.) (1994) Cost-Benefit Analysis, Cambridge University Press.
9. Miller, T., (2000) Variations between Countries in Values of Statistical Life. *Journal of Transport Economics and Policy*, 34:2, 169-188.
10. Nellthorp, J., Sansom, T., Bickel, P., Doll, C., Lindberg, G., (2001) Valuation Conventions for UNITE, Tech. rep., UNITE (UNIfication of accounts and marginal costs for Transport Efficiency).
11. Rosenblatt, M., (1952) Remarks on multivariate transformation. *Annals of mathematical statistics*, 23, 470-472.
12. Viscusi, W., Aldy, J., (2003) The value of a statistical life: a critical review of market estimates throughout the world. *NBER Working Papers*, No. 9487.
13. Zamparini, L., Reggiani, A., (2007a) Meta-Analysis and the value of travel time savings: a transatlantic perspective in passenger transport, *Networks and Spatial Economics*, 7, 377-396.
14. Zamparini, L., Reggiani, A., (2007b) The value of travel time in passenger

and freight transport: an overview. Policy analysis of transport networks, ed. by M. van Geenhuizen, A. Reggiani, and P. Rietveld, Ashgate, Aldershot, chap. 8, 145-161.

15. Zellner, A. (1971) An Introduction to Bayesian Inference in Econometrics. New York: John Wiley & Sons.

Figures

Figure 1: Truncated Predictive Distribution of VTTS
(Trip purpose: bussiness)

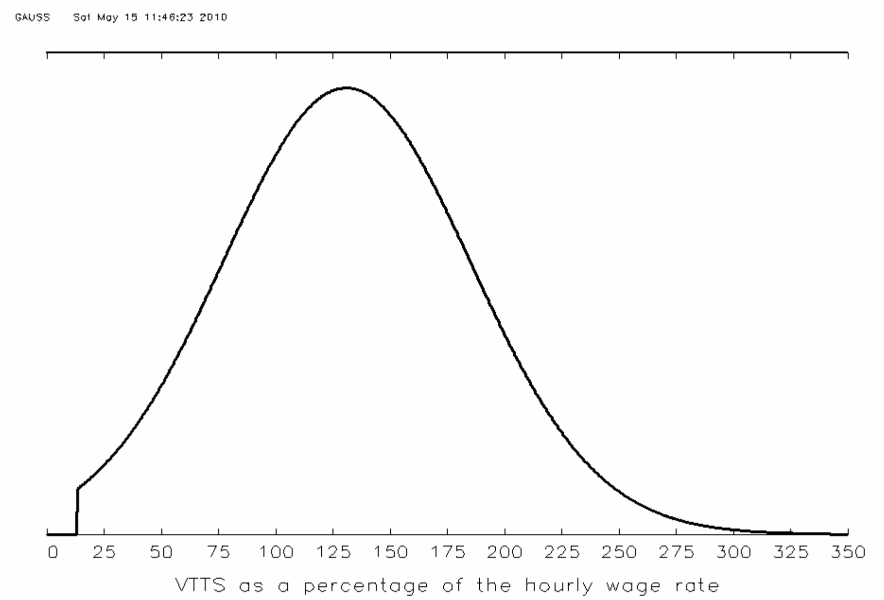
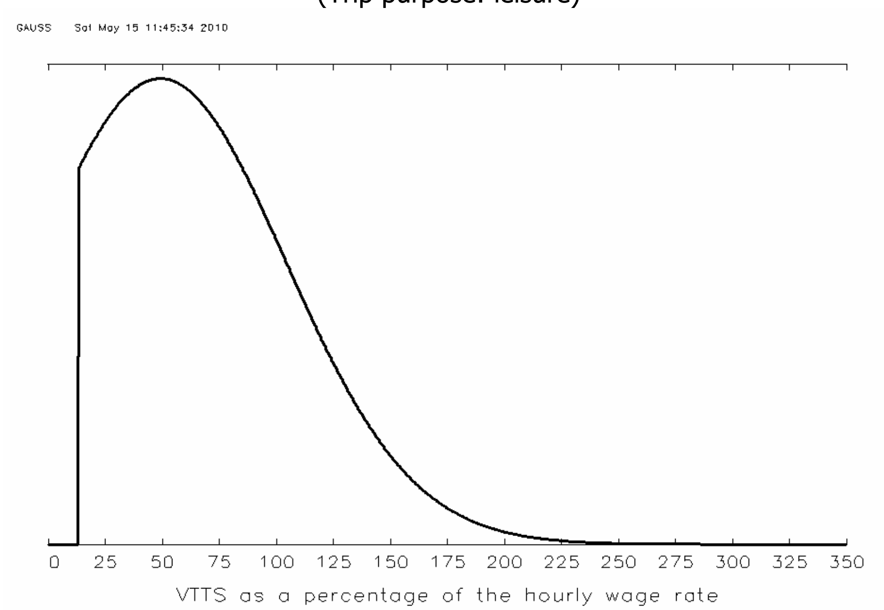


Figure 2: Truncated Predictive Distribution of VTTS
(Trip purpose: leisure)



Tables

Table 1: Descriptive Statistics for per capita GDP.

	Per capita GDP (in thousands of 2000 USD)
Mean	18.65
Median	19.03
Maximum	31.38
Minimum	10.13
Standard Deviation	4.52
Observations	90

Table 2: Project Characteristics.

Country	Spain
Year	2008
Mode	Train
Trip Purpose	Business
Per capita GDP	29,960 USD

Table 3: Variable Definitions and Sources.

Variable	Source	Definition
VTTS	Zamparini and Reggiani (2007a)	Value of Travel Time Savings as a percentage of the hourly wage rate.
GDP	PWT 6.2	GDP per capita (thousands 2000 US dollars at PPP)
Region	Zamparini and Reggiani (2007a)	Geographical region of each of the 90 studies. North Europe (44%); Centre-South Europe (28%); Noth America (20%); Australia (8%)
Year	Zamparini and Reggiani (2007a)	Year in which the study was conducted. 0 if before 1995; 1 if after 1995
Trip Purpose	Zamparini and Reggiani (2007a)	Trip purpose. business (29%); commuting (45%); others (26%)
Mode	Zamparini and Reggiani (2007a)	Mode of transport. airplane (5%); bus (9%); car (77%); train (9%).